# 7th Grade Team Contest

IMSA *Mu Alpha Theta*

February 22, 2023

1. An isosceles triangle has base angles equal to 2x + 30 and 5x - 30. What is the remaining angle of the triangle?

2. Define $x \odot y = 3x + 4y - 6$. Define $x \diamond y = 5x - 2y + 7$. Compute $13 \odot (9 \diamond (5 \odot 2))$.

3. There are 13 black socks and 7 white socks in a drawer. What *percentage* of the socks must be taken out of the drawer to *guarantee* that two different-colored socks have been taken out?

4. There are only cellists and violists in a room. 4 percent of the cellists are also violists and 10 percent of the violists are also cellists. There are only two people who are both cellists and violists. How many people total are there in the room?

5. How many of the numbers, from 1 to 9 inclusive, have their 2023rd power end in 3? (That is: how many of $1^{2023}, 2^{2023}, \ldots, 9^{2023}$ end in 3?)

6. If we know that two sides of a triangle have lengths 3 and 6, what is the sum of all whole numbers that could be the length of the third side?

7. How many integers between 1 and 10000 have exactly two 1's when written out (in standard base-10 notation)?

8. Suppose it is 8 a.m. and John is starting his run. Sam sees him running and also wants to run. If John runs at 15 kilometers per hour and Sam starts running 20 minutes later at 18 kilometers per hour, what time is it when Sam catches up to John?

9. There is an integer that satisfies the equation $\sqrt{210 - n} = 210 - n^2$. Compute the value of n.

10. Aashima is writing the word PROBLEM on the board. But she is really bad a spelling, so she forgets a vowel and swaps two of the consonants. How many different misspellings are there that Aashima could have written?

11. A company wants to buy a piece of land for their new office. The land is owned by a crazy mathematician who tells them the following: the price (in dollars) is less than a million dollars and it is the largest number that starts with a 7, ends with a 2, has three 3's in it, and is divisible by both 8 and 9. How much does the property cost?

12. The diagram below left shows a map of the neighborhood, with streets running east-west and north-south. You wish to go from your house H to the store S by traveling only north or east. How many different routes could you take?
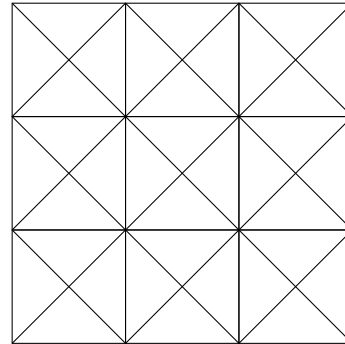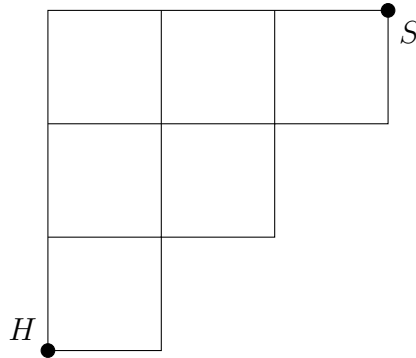
13. How many triangles, of any size, are there whose sides are in the grid drawn above right?

14. The sum of 14 consecutive even numbers is 70. Find the product of the smallest and largest of these numbers.

15. The math club collected contributions for the most valuable mathlete prize. They collected $5.00 in pennies, nickels, dimes, and quarters. They have 50% more quarters than dimes, half as many pennies as dimes, and 4/5 as many nickels as pennies. How many nickels did they collect?

16. Start writing the numbers: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 . . . . Note that the 17th digit you wrote was a 3. If you continue long enough, what will be the 2023rd digit you write?

17. In a survey, 38 families have a pet cat and 32 of them have pet dogs. Just as many families had no pets at all as there were families that had both cats and dogs. How many families were in the survey?

18. Two poles are standing on level ground. One is 20 feet tall, the other is 30 feet tall. There is a wire connecting the top of each pole to the bottom of the other. How high off the ground do these wires cross?

19. Alice and Bob play a game. They take turns flipping a coin, Alice going first. The first person to flip heads wins. What is the probability that Alice wins?

20. Think about an $8 \times 8 \times 8$ 3-dimensional chess board. It has $8 \cdot 8 \cdot 8 = 512$ small "cubelets" in it. A cat and a mouse start in opposite cubelets, as far apart as possible. Each second both animals move to any adjacent cubelet, left or right, up or down, or back or forth. To the nearest tenth, compute the probability that the cat will and mouse will end up in the same cubelet within the first 512 seconds.