Creative Thinking

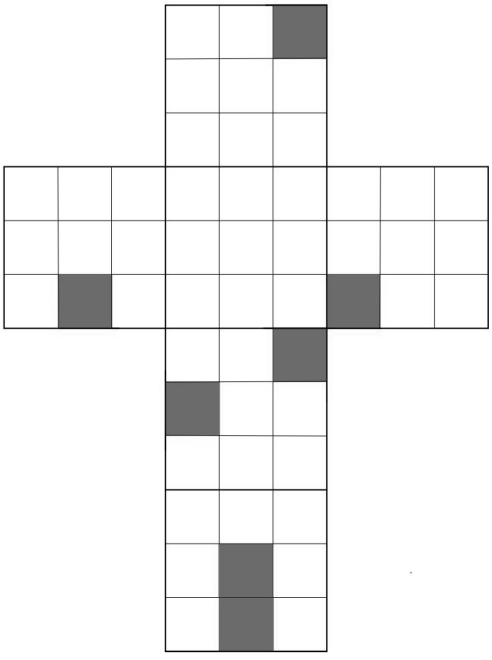
IMSA *Mu Alpha Theta*

March 2, 2022

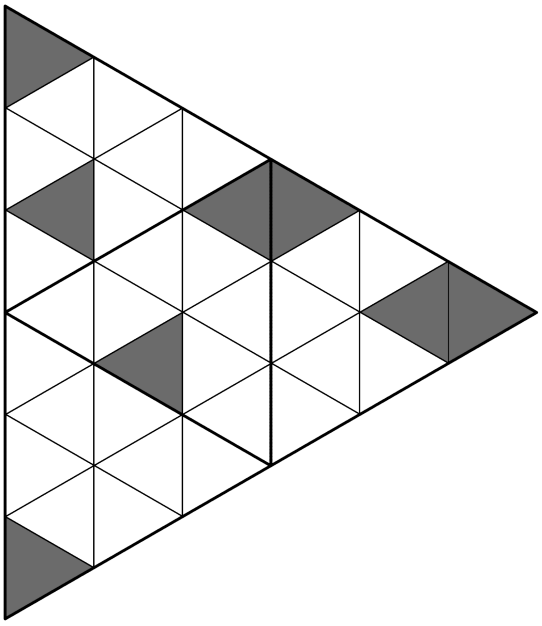
1. Nets

a) You're given the net (the pattern obtained when a three-dimensional figure is laid out flat) of a $3 \times 3 \times 3$ cube below. When the shape is in its three-dimensional configuration, if a $1 \times 1 \times 1$ cube of the larger $3 \times 3 \times 3$ cube has one or more sides shaded gray, the unit cube is removed from the larger cube. What is the final volume of the $3 \times 3 \times 3$ cube shown below after the unit cubes are removed?

b) You're now given a net of a tetrahedron made up of smaller, equal sized, equilateral triangles. When the shape is in its three dimensional configuration, any small tetrahedrons, with volume 1, that are created by the triangles are removed from the larger tetrahedron. What is the final volume of the tetrahedron with the smaller tetrahedrons removed?



Net of Cube Figure for Problem 1a



Net of Tetrahedron Figure for Problem 1b

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2. Greedy Algorithms

a) Terms in the *Fibonacci sequence* are calculated by summing the two preceding terms. The first few numbers in the Fibonacci sequence are $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89$, and 144. *Zeckendorf's Representation Theorem* states that every positive integer can be represented as a unique sum of distinct, nonconsecutive Fibonacci numbers. Some examples of Zeckendorf representations include $75 = 2 + 5 + 13 + 55$, $50 = 3 + 13 + 34$, and $25 = 1 + 3 + 21$. Find the Zeckendorf representation of 321.

b) Your are an activity director at a local youth center. Today, you have the opportunity to host the following activities for the kids at the center:

Activity Number	Activity Name	Start Time	End Time
1	Swimming	9:30am	1:30pm
2	Dance	12:45pm	1:30pm
3	Movies	8:00am	10:00am
4	Parachute	1:00pm	4:15pm
5	Laser Tag	9:00am	12:15pm
6	Nap Time	4:15pm	5:00pm
7	Rock Climbing	12:00pm	2:30pm
8	Freeze Tag	12:00pm	12:45pm
9	Theme Park	11:30am	1:15pm
10	Yoga	3:00pm	4:30pm
11	Simon Says	2:00pm	4:00pm
12	Three-Legged Race	3:45pm	5:30pm
13	Pizza Party	9:45am	12:15pm
14	Story Time	8:30am	9:30am
15	Volleyball	9:15am	10:30am

You cannot schedule two activities if their time frame is overlapping. If your goal is to maximize the number of activities offered today, what activities should you use? Report your answer as a list of numbers used to label the activities. For example, If you wanted to schedule Swimming and Bowling, you would report $1, 6$.

3. Playing with Logic

a) Caroline is playing a game with Alice and Bob. She writes a non-negative integer on both of their heads, so that they both know the other's number, but not their own. Caroline then writes two numbers on a board they can both see; one of these numbers is the sum of the numbers written on their heads. In the first round, Caroline asks Alice if she knows her number, in the second she asks Bob, in the third she asks Alice again, etc. until one of them answers "Yes." Caroline writes 1 on both of their heads and writes 2 and 3 on the board. If both Alice and Bob are perfectly honest and intelligent, the game ends on which round?

b) Alice, Bob, and Caroline play the same game again, only this time Caroline writes a 2 on Alice's head, a 3 on Bob's head, and the numbers 3 and 5 on the board. The game ends on which round?

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4. Numbers in ASMI

In the world of ASMI, numbers work a little bit differently. The symbols $\odot$, ∇, $\heartsuit$, $\diamondsuit$, $\bowtie$, and $\triangle$ each represents a single digit, while the $+$, $-$, $\times$ and $=$ have their usual arithmetic meanings. Additionally, there is a positive integer, b, so that if two digits x and y are written next to each other, as in xy, the value of this is $bx + y$. For example, if $\bowtie$ represents 8 and $\odot$ represents 9, and $b = 4$ then $\bowtie \odot$ represents $4 \times 8 + 9 = 41$. Part of this problem will be to find the correct values of the digits and value of b. Some clues about numbers in ASMI are given below:

1. $\odot + \nabla = \nabla$
2. $\nabla \times \heartsuit = \heartsuit$
3. $\diamondsuit + \nabla = \nabla\odot$
4. $\diamondsuit - \nabla - \nabla - \nabla = \nabla + \nabla$
5. $\heartsuit \times \bowtie = \nabla\odot$
6. $\diamondsuit + \diamondsuit = \nabla\triangle$
7. $\nabla\nabla + \nabla = \nabla\heartsuit$

a) What is $\nabla \times (\bowtie \times \bowtie - \diamondsuit)$ written in ASMI notation (using the six symbols from above)?

b) What is $\triangle + \diamondsuit + \nabla\nabla$ written in ASMI notation?