7^{th} Grade Individual Contest

IMSA *Mu Alpha Theta*

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1. The product of four distinct natural numbers is 100. What is their sum?

2. Jim has a lot of coins! He has only nickels, dimes, and quarters, and he has the same number of each kind of coin. In total, Jim has 6 dollars in change. How many coins does Jim have?

3. Joe is driving from his house to his job. He first travels north for 7 miles, then west for 3 miles. He goes north again for 6 more miles, east for 11 miles, and finally north again for 2 miles to arrive to his workplace. If a line segment was drawn straight from Joe's house to his workplace, how long (in miles) would that segment be?

4. How many positive integers n are there that satisfy $n \leq 20$ and $\frac{4n}{6} - 5$ is a positive integer?

5. We can write $1.0\overline{5} = 1.05555\ldots$, with 5's repeating forever, as an improper fraction $\frac{p}{q}$ where p and q are positive whole numbers that share no common factors. Determine $p + q$.

6. Define $a \oplus b$ to mean $a(3a+b)$ and $a \otimes b = \dfrac{b^2 + 2a}{b} \oplus 2$. What is the value of $9 \oplus ((2 \oplus 3) \otimes 4)$?

7. Shiqi has lost her marbles. These marbles can be red, blue, or yellow. If $\frac{2}{3}$ are red, 25% are blue, and 17 are yellow, how many marbles has Shiqi lost?

8. Solve the following equation for x:

$$\frac{x^2 - 6}{x - 3} = x.$$

9. 10 years ago, Andy was x times as old as his brother Bruce. Now, Andy is $x - 2$ times as old as Bruce. Given that the sum of their current ages is 45, determine the value of x.

10. Gerald owns a restaurant. Gerald is also a scoundrel, so he often dilutes his high-quality 94% pure orange juice with cheap low-quality 42% pure orange juice to increase his profits. From years of experimenting on his customer's taste buds, Gerald has discovered that customers start getting suspicious if the orange juice drops below 67% pure. To get through a day of sales, Gerald needs to prepare 60 gallons of orange juice. What is the largest whole number of gallons of low quality orange juice that Gerald can squeeze into the mixture before the customers get suspicious?

11. Yina gave her students a math test. Two students tied for the highest score. She finds that her students' average score is 85% with a range of 60% in the scores. Checking her work, Yina discovers that she had made a mistake; instead of counting the highest score twice she had counted the lowest score twice when computing the average. Find the correct average of the students' scores.

12. Determine the smallest positive integer n such that $n = ab$ and $2a^2 = 3b^3$ where a and b are also integers.

13. A right triangle has an area of 9 and a hypotenuse that is 8 units long. What is the perimeter of the triangle?

14. In parallelogram $ABCD$, a line segment is drawn from point B perpendicular to the diagonal $\overline{AC}$ at point E. The distance between lines $\overline{AB}$ and $\overline{CD}$ is 6. The length of $\overline{AB}$ is 12 and the length of $\overline{AC}$ is 9. Find the length of segment $\overline{BE}$.

15. The set of all lines that have a slope equal to their y-intercept intersect at a common point, (a, b). Compute $|a + b|$.

16. A square is inscribed in a circle and this circle is inscribed in an equilateral triangle. Compute the ratio of the area of the triangle to the area of the square.